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[34/A-14] **Seat No.:_____** No of printed pages : 3

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - VI) EXAMINATION - 2018
Friday , 6th April , 2018
MATHEMATICS : US06CMTH06
(MECHANICS - 2)

Time : 10:00 a.m. to 1:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) Linear momentum in polar coordinate system is $\bar{p}$ =.....

(a) $(m\dot{r}\ ,\ mr\dot{\theta})$ (b) $(m\dot{r}\ ,\ m\dot{r}\theta)$ (c) $(m\dot{r}\ ,\ mr\theta)$ (d) $(m\dot{\theta}\ ,\ mr\theta)$

(2) The reversed effective forces and the real forces together give .....

(a) potential energy (b) kinetic energy (c) force (d) statical equilibrium

(3) Angular momentum of a particle about a point $A(2,3)$ is .....

(a) $m((x-2)\dot{x}-(y-3)\dot{y})$ (b) $m((x-2)\dot{y}+(y-3)\dot{x})$ (c) $m((x-2)\dot{y}-(y-3)\dot{x})$ (d) $m((x-2)\ddot{y}-(y-3)\ddot{x})$

(4) The equation of motion of a projectile with resistance for the forces vertically is given by .....

(a) $m\ddot{x}+R\cos\theta=0$ (b) $m\ddot{y}+R\sin\theta+mg=0$ (c) $\ddot{y}+R\sin\theta+mg=0$ (d) $m\ddot{y}+R\sin\theta+g=0$

(5) The equation of motion of projectile is .....

(a) $y=x\tan\alpha-\dfrac{gx^2}{2u^2\cos^2\alpha}$ (b) $y=u\sin\alpha\, t-\dfrac{gt^2}{2}$ (c) $y=x\tan\alpha+\dfrac{gx^2}{2u^2\cos^2\alpha}$ (d) both a and b

(6) The path of projectile is .....

(a) circle (b) ellipse (c) parabola (d) hyperbola

(7) Moment of inertia of rod depends on ..... of rod .

(a) b and c both (b) mass (c) length (d) radius

(8) The squares of the periodic times of the planets are proportional to the ..... of the semi major axis of their orbits .

(a) cube roots (b) square roots (c) squares (d) cubes

(9) The coefficient of restitution e is always .....

(a) 1 (b) ≥ 0 (c) < 1 (d) > 0

(10) A sphere with elasticity 0.5 collide directly to another identical sphere at rest , their velocities after impact are in the ratio .....
(a) 3 : 1 (b) 1 : 3 (c) 1 : 2 (d) 2 : 1

Que.2 Answer the following (Any ten) 20

(1) Define : Linear momentum , Angular momentum about a point , Linear momentum of system , Angular momentum of system .

(2) If force acting on particle is perpendicular to it's velocity then prove that speed of particle is constant.

(3) Find Angular momentum of particle in polar coordinate system at $(r,0)$.

(4) If R is the horizontal range and H is the greatest height attained by the projectile then prove that the initial velocity of projectile is given by $\sqrt{2g\left(H+\frac{R^2}{16H}\right)}$.

(5) A particle is projected upward in the direction of making an angle 60° with the horizontal. Show that its velocity at maximum height is half of its initial velocity (Neglect resistance of air) .

(6) A particle just clear a wall of height 'b', at a distance 'a' and and strikes the ground at a distance 'c', from the point of projection. Prove that the angle of projection is given by, $\alpha = \tan^{-1}\left(\frac{bc}{ac-a^2}\right)$.

(7) In a motion under a central force , prove that the areal velocity is constant .

(8) Find the law of force towards the pole for the curve described by $r = ae^{\theta\cot\alpha}$.

(9) By using theorem of perpendicular axes find moment of inertia of a rectangular plate of mass m and edges of lengths $2a$ and $2b$ about a line through its center perpendicular to its plane .

(10) Find the equation of motion of flywheel .

(11) State and prove Principle of angular momentum with respect to mass center relative to impulsive force .

(12) Find workdone by the impulsive force.

Que.3 (a) State and prove principle of conservation of energy for system of particle . 5

(b) Obtain useful forms of equations of motion of a particle. 5

OR

Que.3 (c) The rate of change of angular momentum of a system relative to the mass center is equal to the moment of the external forces about the mass center. 5

(d) Verify the principle of conservation of energy, if a particle of mass m (i) falling vertically downward under the force of gravity (ii) slides down on a smooth inclined plane starting from the rest. 5

Que.4 (a) Obtain the equation of motion of projectile with resistance in the form

$$x = x_0 + u_x\,t - \frac{1}{2}\,\phi\,u_x\,t^2 \quad ; \quad y = y_0 + u_y\,t - \frac{1}{2}\,g\,t^2 - \frac{1}{2}\,\phi\,u_y\,t^2\left(1-\frac{gt}{3\,u_y}\right).$$ 6

(b) A particle of mass m is projected vertically upward in medium for which resistance R is mk^2v^2 .If the initial velocity is v_0 then show that the particle returns to the point of projection with velocity v_1 such that $v_1^2 = \frac{gv_0^2}{g+kv_0^2}$. 4

OR

Que.4 (c) For a particle, moving with resistance which is independent of height, prove that $\frac{1}{v}\frac{dv}{d\psi} = \tan h\psi + \phi(v)$. 5

(d) A particle of mass m is projected in a vertical plane through the point of projection with velocity v_0 in the direction making an angle α with the horizontal axis .Show that the path of projectile is parabola .Find its vertex ,focus and equation of directrix . 5

Que.5 (a) State and prove the theorem of *KÖNIG*. 4

(b) Obtain differential equation of central orbit . 4

(c) State the laws of the inverse square . 2

OR

Que.5 (d) In usual notation prove that the semi latus rectum and the eccentricity are given by $l = \frac{h^2}{\mu}$; $e = \sqrt{1 + \frac{2Eh^2}{\mu^2}}$ respectively . 7

(e) By using theorem of parallel axes find moment of inertia of a rod of mass m and length $2a$ about a line through one end perpendicular to the rod . 3

Que.6 (a) A series of n - elastic spheres of masses $1, e, e^2, \ldots, e^{n-1}$ are at rest separated by intervals with their centre in the straight line .The first made to collide directly to t he second ball with velocity u . Show that the first $n-1$ spheres will move the same velocity $(1-e)u$ and the last with velocity u . Also prove that kinetic energy of the system is $\frac{1}{2}(1 - e + e^n)u^2$. 6

(b) A particle falls from height h on a horizontal plane and rebounds continuously .Show that whole time before particle comes to rest is $\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$. 4

OR

Que.6 (c) Obtain the loss of kinetic energy due to impact of two spheres of masses m_1 and m_2 with velocities u_1 and u_2 relative to the mass center . 4

(d) State and prove Principle of linear momentum relative to impulsive force . 3

(e) A sphere collides directly to an equal sphere which is at rest ,show that the fraction $\frac{1}{2}(1-e^2)$ of the original kinetic energy is lost during the impact . 3