SEAT No.________

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[79] SARDAR PATEL UNIVERSITY

B.Sc. SEM- V EXAMINATION (NC)

10-04-2018, Tuesday

02.00 p.m. to 05.00 p.m.

US05CMTH02

(Real Analysis-II)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) A positive term series $\sum \frac{1}{n^p}$ is convergent iff
(a) $p = 1$ (b) $p < 1$ (c) $p > 1$ (d) $p = 0$

(2) $\lim_{(x,y)\to(0,0)} (x + y) = ...$
(a) 0 (b) (0,0) (c) -1 (d) not exists

(3) The Range of sequence is always .... set
(a) empty set (b) infinite set (c) non-empty set (d) none of these

(4) A function is continuous if.....
(a) $\lim_{(x,y)\to(a,b)} f(x, y) \neq f(a, b)$ (b) $\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$
(c) $\lim_{(x,y)\to(a,b)} f(x, y) = f(0, 0)$ (d) none of these

(5) $\lim_{x\to1} \lim_{y\to-1} \frac{-4x^3y^2}{x^2 + y^2} =$
(a) -2 (b) 3 (c) 2 (d) 1

(6) A series $\sum u_n$ is convergent then ...
(a) $\lim_{n\to\infty} u_n = 1$ (b) $\lim_{n\to\infty} u_n \neq 0$ (c) $\lim_{n\to\infty} u_n = 0$
(d) $\lim_{n\to\infty} u_n$ does not exists

(7) If $f(x, y) = 2x^3 + 3y^2$ then $f_{yyy} = ...$
(a) $6y$ (b) $2x^3$ (c) 6 (d) 0

(8) The function $f(x, y)$ has an extreme value at point (a, b) if...
(a) $f(x, y) = f(a, b)$ (b) $f_x(a, b) = 0 = f_y(a, b)$
(c) $f_x(a, b) \neq 0$ (d) $f_x(a, b) > 0$

(9) A stationary point is called saddle point of function f if it is...
(a) extreme point (b) not an extreme point (c) $f_x(a, b) = 0$
(d) none of these

(10) A sequence $\{1, 1, 2, 1, 3, 1, 4, 1, \dots\}$ has ...... as a limit point.
(a) 1 (b) 2 (c) 0 (d) not exists

Q.2 Attempt any ten in short: [20]

(1) State Maclaurin's theorem.
(2) Define limit and repeated limit of a function of two variables.
(3) Define Neighbourhood of a point of two variable.
(4) Prove that $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$ is not convergent. [PTO]

(5) Prove that $\lim_{(x,y)\to(0,0)} \frac{\sin^{-1}(xy-2)}{\tan^{-1}(3xy-6)} = \frac{1}{3}$.

(6) State comparison test for second type for series.

(7) State necessary condition for $f(x,y)$ to have an extreme value at (a,b).

(8) Show that every convergent sequence has a unique limit.

(9) Prove that $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$ does not exists.

(10) Test for convergence the series $\sum \frac{1}{n^{1+\frac{1}{n}}}$.

(11) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

(12) If $\sum u_n = u$ and $\sum v_n = v$, then prove that $\sum(u_n + v_n) = u + v$.

Q.3(a) State and prove Bolzano-Weierstrass theorem for sequences. [5]

(b) Show that every convergent sequence is bounded and has a unique limit. [5]

OR

Q.3(c) Show that the necessary and sufficient condition for the convergence of a sequence $\{S_n\}$ is for each $\epsilon > 0$, there exist a positive integer m such that $|s_{n+p} - s_n| < \epsilon, \forall n \geq m \wedge p \geq 1$. [5]

(d) Prove that the set of limit points of a bounded sequence has the greatest and the least member. [5]

Q.4(a) If $\sum u_n$ and $\sum v_n$ are two positive term series, and $k \neq 0$, a fixed positive number (independent of n) and there exists a positive integer m such that $u_n \leq kv_n$, for all $n \geq m$, then Prove that [5]

(i) $\sum u_n$ is convergent if $\sum v_n$ is convergent and

(ii) $\sum v_n$ is divergent if $\sum u_n$ is divergent.

(b) State and Prove D'Alembert's Ratio test. [5]

OR

Q.4(c) State and prove Cauchy's root test. [5]

(d) Show that the series $\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \cdots + \frac{1}{(\log n)^p} + \ldots$, diverges for $p > 0$. [5]

Q.5(a) Prove that, by the transformation $u = x - ct$, $v = x + ct$, the partial differential equation $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ reduces to $\frac{\partial^2 z}{\partial u \partial y} = 0$. [5]

(b) If V is a function of two variables x and y and $x = r\cos\theta$, $y = r\sin\theta$, then prove that [5]

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r}\frac{\partial V}{\partial r}.$$

OR

Q.5(c) Show that for the functions $f(x,y) = \begin{cases} x\sin(\frac{1}{y}) + y\sin(\frac{1}{x}), & xy \neq 0 \\ 0, & xy = 0 \end{cases}$ [5]

limit exist at the origin but the repeated limits do not.

(d) Show that the function

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is continuous at the origin. [5]

Q.6(a) State and prove Taylor's theorem. [5]

(b) Find maxima and minima of the function

$$x^3 + y^3 - 3x - 12y + 20.$$

[5]

OR

Q.6(c) Investigate the maxima and minima of the function [5]

$$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy.$$

(d) Prove that for $0 < \theta < 1$,

$$e^{ax}\sin by = by + abxy + \frac{1}{6}[(a^3x^3 - 3ab^2xy^2)\sin b\theta y + (3a^2bx^2y - b^3y^3)\cos b\theta y]e^{a\theta x}.$$

[5]

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