



(Master of Science) (Mathematics)  
(M.Sc.) (Mathematics) Semester (II)

|                             |            |                     |                       |
|-----------------------------|------------|---------------------|-----------------------|
| Course Code                 | PS02CMTH54 | Title of the Course | Functional Analysis-I |
| Total Credits of the Course | 04         | Hours per Week      | 04                    |

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| Course Objectives: | 1. To study finite and infinite-dimensional vector spaces equipped with an inner product.<br>2. To give a working knowledge and ideas of the theory of Hilbert spaces, bounded linear operators and their spectra, duals, and adjoint. |
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| Course Content |   |                |
|----------------|---|----------------|
| Unit           | Description   | Weightage* (%) |
| 1.             | Inner product spaces, normed linear spaces, examples of inner product spaces, Polarization identity, Schwarz inequality, parallelogram law, uniform convexity of the norm induced by inner product, orthonormal sets, Pythagoras theorem, Gram-Schmidt orthonormalization, Bessel's inequality, Riesz-Fischer theorem. Hilbert spaces, orthonormal basis, characterization of orthonormal basis, separable Hilbert spaces.          | 25             |
| 2.             | Uniqueness of best approximation from a convex subset of inner product space to a point, orthogonality and best approximation, existence and uniqueness of best approximation from a convex subset of a Hilbert space to a point, continuity of a linear mapping, projection theorem and Riesz representation theorem, weak convergence and weak boundedness.   | 25             |
| 3.             | Bounded operators, equivalence of boundedness and continuity of an operator, boundedness of the operator associated to an infinite matrix, adjoint of a bounded operator, properties of adjoint, relations between zero space and the range of operators, normal, unitary and self-adjoint operators, examples, characterizations and results pertaining to these operators, positive operators and generalized Schwarz inequality. | 25             |
| 4.             | Spectrum, eigenspectrum, approximate eigenspectrum, definition and characterization, spectrum of a normal operator, examples, numerical range, relations of numerical range and different spectra, spectral theorem for a normal/self-adjoint operator on a finite dimensional Hilbert space, compact operators, properties of compact operators.   | 25             |





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| Teaching-Learning Methodology | Classroom teaching, problem solving, independent reading |
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| Evaluation Pattern |  |           |
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| Sr. No.            | Details of the Evaluation  | Weightage |
| 1.                 | Internal Written / Practical Examination (As per CBCS R.6.8.3)   | 15%       |
| 2.                 | Internal Continuous Assessment in the form of Practical, Viva-voce, Quizzes, Seminars, Assignments, Attendance (As per CBCS R.6.8.3) | 15%       |
| 3.                 | University Examination   | 70%       |

| Course Outcomes: Having completed this course, the learner will be able to |  |
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| 1.   | appreciate how functional analysis unifies the ideas of vector spaces, metric spaces and topological spaces, and complex analysis.                                     |
| 2.   | understand and apply the fundamental results in the theory of Hilbert spaces including the Riesz-representation theorem, Gram-Schmidt orthonormalization process, etc. |
| 3.   | understand the fundamentals of the spectral theory.  |
| 4.   | prepared for an advanced course in Functional Analysis and Operator theory.  |

| Suggested References: |   |
|-----------------------|---|
| Sr. No.               | References  |
| 1.                    | Limaye B.V., Functional Analysis, New Age International Publ. Ltd., New Delhi, 1996.        |
| 2.                    | Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill Co., Tokyo, 1963.  |
| 3.                    | Thumbar Nair, Functional Analysis: A First Course, Prentice-Hall of India, New Delhi, 2002. |

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| On-line resources to be used if available as reference material |
| On-line Resources   |

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