ON A NEW SEQUENCE OF FUNCTIONS DEFINED BY A
GENERALIZED RODRIGUES FORMULA

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ABSTRACT

In present paper, an attempt is made to provide an elegant unification of several classes of polynomials. Introduced sequence of the functions \(V_n^{(a,b)}(x,a,k,s)/n=0,1,2,...\) by means of generalized Rodrigues formula (7), which involves Mittag-Leffler function \(E_\alpha(z)\) and other two similar kind of class of polynomials (5), (6). Some generating relations and finite summation formulae have also been obtained for (7).

Keywords: Mittag-Leffler function, generating relations, finite summation formulae.


INTRODUCTION:

In 1979, Srivastava and Singh [7] introduced a general sequence of functions \(V_n^{(a,b)}(x,a,k,s)\) by employing the operator \(x^{(s+x)D}\) to \((5),(6)\) and \(7\) respectively as:

\[V_n^{(a,b)}(x,a,k,s) = \frac{1}{n!} x^{(s+x)D}\]

In the present paper, the generating relations and finite summation formulae obtained for sequence of functions \(7\) as these are obviously more powerful sequence of functions than \(5\) and \(6\). The technique discussed in this paper will certainly apply for sequence of functions \(5\) and \(6\).

To obtain generating relations and finite summation formulae, the properties of the differential operators \(\theta \equiv x^{(s+x)D}\) and \(\theta_1 \equiv x^{(1+x)D}\) where \(D_x^{\alpha}\) used on the based of work (Mittal [3], Patil and Thakare [5]).

In the fourth section, the relations between \(7\) with some well-known polynomials \(9\) and \(10\) also have been discussed.

Hermit polynomials (Rainville [6]) defined as:

\[H_n(x) = (-1)^n \exp(x^2)D^n[\exp(-x^2)]\]

Konhauser polynomials of first kind (Srivastava [9]) defined as:

\[Y_n(x;k) = \frac{\chi^{n-1} e^{-x^2} D^n[\chi^{(s+x)D}]}{k^n n!} \]

GENERATING RELATIONS:

We obtained some generating relations of \(7\) as,

\[\sum_{n=0}^{\infty} x^{-n} V_n^{(a,b)}(x,a,k,s) \Gamma(n) = \frac{\Gamma(z) \theta^n}{\Gamma[z,\Gamma^{(1+a)}(\theta^n)]} \]

\[\sum_{n=0}^{\infty} x^{-n} V_n^{(a,b)}(x,a,k,s) \Gamma(n) = \frac{\Gamma[z,\Gamma^{(1+a)}(\theta^n)]}{\Gamma[z,\Gamma^{(1+a)}(\theta^n)]} \]
\[
\sum_{n=0}^{\infty} \left( \sum_{k=0}^{\infty} x^{a_k} \right) \exp^{-\gamma} = \left( 1 - x \right)^{-\gamma} E_{\nu}[\gamma (x, k)]
\]

Proof of (11):
From (7), we consider
\[
\sum_{n=0}^{\infty} \left[ \sum_{x=0}^{\infty} \left( \sum_{k=0}^{\infty} x^{a_k} \right) t^x \right] E_{\nu}[\gamma (x, k)]
\]
above equation reduces to
\[
\sum_{n=0}^{\infty} \left[ \sum_{x=0}^{\infty} \left( \sum_{k=0}^{\infty} x^{a_k} \right) t^x \right] E_{\nu}[\gamma (x, k)]
\]
and replacing \( t \) by \( \gamma t^x \), which gives (11).

Proof of (12):
From (7), we consider
\[
\sum_{n=0}^{\infty} x^{-\alpha \beta} V_{\nu}^{(a, \beta)}(x; a, k, s) t^x
\]
\[
= x^{-\alpha \beta} E_{\nu}[\gamma (x, k)]
\]
and simplifying the above equation, we get
\[
\sum_{n=0}^{\infty} x^{-\alpha \beta} V_{\nu}^{(a, \beta)}(x; a, k, s) t^x = \frac{1}{E_{\nu}[\gamma (x, k)]}
\]
which proves (12).

Proof of (13):
writing (7) as
\[
\theta^a \cdot x^b E_{\nu}[\gamma (x, k, s)]
\]
\[
= n! \cdot x^b E_{\nu}[\gamma (x, k, s)]
\]
or
\[
ed^a \cdot \theta^a \cdot x^b E_{\nu}[\gamma (x, k, s)]
\]
\[
= n! \cdot e^{a \cdot \theta^a} \cdot x^b E_{\nu}[\gamma (x, k, s)]
\]
above equation can be written as
\[
\sum_{n=0}^{\infty} \frac{x^b}{n!} \cdot n^{-\alpha \beta} \cdot x^b E_{\nu}[\gamma (x, k, s)]
\]
\[
= n! \cdot x^b \left( 1 - ax^{\alpha \beta} \right) \frac{1}{E_{\nu}[\gamma (x, k, s)]}
\]
\[
= \frac{1}{E_{\nu}[\gamma (x, k, s)]}
\]
and above expression reduces to
\[
\sum_{n=0}^{\infty} \left( \frac{m+n}{n} \right) V_{\nu}^{(a, \beta)}(x; a, k, s) t^x
\]
\[
= (1 - ax^{\alpha \beta}) \frac{1}{E_{\nu}[\gamma (x, k, s)]}
\]
replacing \( t \) by \( \gamma t^x \), which leads to (13).

**FINITE SUMMATION FORMULAE:**
We obtained two finite sum formulae for (7) as
\[
V_{\nu}^{(a, \beta)}(x; a, k, s) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot \left( \sum_{x=0}^{\infty} \left( \sum_{k=0}^{\infty} x^{a_k} \right) \right) E_{\nu}[\gamma (x, k)]
\]
(14)

Proof of (14):
We can write (7) as
\[
V_{\nu}^{(a, \beta)}(x; a, k, s) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot E_{\nu}[\gamma (x, k)]
\]
we get,
\[
V_{\nu}^{(a, \beta)}(x; a, k, s) = \frac{1}{n} \cdot \frac{x^n}{n!} \cdot E_{\nu}[\gamma (x, k)]
\]
which yields
\[
V_{\nu}^{(a, \beta)}(x; a, k, s) = \frac{1}{n} \cdot \frac{x^n}{n!} \cdot E_{\nu}[\gamma (x, k)]
\]
(15)

Proof of (15):
From (15) consider,
\[
\sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot E_{\nu}[\gamma (x, k)]
\]
above equation reduces to,
\[
\sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot \left( 1 - ax^{\alpha \beta} \right) \frac{1}{E_{\nu}[\gamma (x, k)]}
\]

use of (17) and (16), which immediately leads (14).

Proof of (15):
From (7) we consider,
\[
\sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot E_{\nu}[\gamma (x, k)]
\]
above equation reduces to,
\[
\sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot \left( 1 - ax^{\alpha \beta} \right) \frac{1}{E_{\nu}[\gamma (x, k)]}
\]
which yields
\[
\sum_{n=0}^{\infty} V_{m}^{(s)}(x,a,k,s) \frac{x^n}{n!} = \frac{(1-ax^t)^{t-1}}{E_a[p(x)]} \sum_{n=0}^{m} \frac{(-1)^n}{m!} \binom{m}{n} (ax^t)^n \times \\
E_a[p(x)] E_a[x(s(1-ax^t))^{\alpha-1}]
\]
by using (11),
\[
\sum_{n=0}^{\infty} V_{m}^{(s)}(x,a,k,s) \frac{x^n}{n!} = \sum_{n=0}^{\infty} \left( \frac{1}{a} \right) \frac{(ax^t)^n}{m!} x^{-s} \times \\
E_a[p(x)] E_a[x(s(1-ax^t))^{\alpha-1}] 
\]
and use of (7), we get (15).

**SPECIAL CASES:**

In this section, we have obtained some special cases and relations of sequence of functions \( V_{m}^{(s)}(x,a,k,s) \), in the connection of (4), (9) and (10): Putting \( a = 1 \) and replacing \( \beta \) by \( \alpha \) in (7) then
\[
V_{m}^{(s)}(x,a,k,s) = V_{m}^{(s)}(x;1,k,s) 
\]
Therefore, we can say that (4) is a particular case of (7).

If \( a = 1 \), replacing \( \beta \) by \( a + 1, \alpha = 1, p_0(x) = p_0(x) = x, \) and \( s = 0, \) then (7) reduces to
\[
V_{m}^{(s)}(x;1,1,0) = x^t V_{m}^{(s)}(x;1) 
\]
and if \( a = 1, \beta = 0, p_0(x) = p_0(x) = x^t, \) and \( s = 0, \) then (7) reduces to
\[
V_{m}^{(s)}(x;-1,2,0) = \frac{(-1)^s}{n!} H_{m}(x) 
\]

**CONCLUSION:**

The new sequence of functions (5), (6) and (7), introduced the in section 1, the results obtained in sections 2, 3 and 4 seems to be new and quite interesting.

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**REFERENCES**