NOTE

MARXIAN MODEL OF GROWTH

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SYMBOLS

$t$ is the time such that a given length of period constitutes a unit of time. Thus, the term, ‘in any given period’ and ‘in any time’ are used synonymously.

$\mathcal{Y}$ is income. The term ‘income’ is used by us to mean net real income (output of the composite commodity after allowing for raw materials used and depreciation of capital in its production) per unit of time. We use the terms ‘income’ and ‘output’ synonymously.

$\mathcal{K}$ is the stock of capital.

$P$ is the working population.

$s$ is the average rate of saving with respect to income.

$w$ is wage per working population.

$w$ is subsistence wage per working population.

THE MODEL

"And while we continuously enjoy pointing out that Marx's prophecies, made more than a hundred years ago, did not come true, we are apt to forget that it is the sign of his genius as an analyst that he foresaw so many relatively important things as accurately as he did"\(^1\).

Marx rejects the classical thesis of historically diminishing returns because of his assumption of sufficiently high rate of techno-

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logical change. In his analysis of 'The General Law' of capital accumulation he assumes work unit per working population to be constant\(^3\). The production function according to Marx is,

\[
\eta = \min \left( \frac{K}{h}, \frac{P}{m_0 L} \right)
\]

where \(h\) and \(m\) are positive constants and

(i) \(u = 0\) — “if we suppose that, all other circumstances remaining the same, the composition of capital also remains constant (i.e., that a definite mass of means of production constantly needs the same mass of labour-power to see it in motion), then the demand for labour and the subsistence-fund of the labourers clearly increase in the same proportion as the capital, and the more rapidly, the more rapidly the capital increases”\(^3\),

or (ii) \(u\) is a negative constant — “But whether condition or consequence, growing extent of the means of production, as compared with the labour-power incorporated with them, is an expression of the growing productiveness of labour”\(^4\),

or (iii) \(u = -u_o \left( t + 1 - \cos t \right)\) such that \(u_o\) is a positive constant — “capital continues growing for a time on its given technical basis, and attracts additional labour-power in proportion to its increase, while at other times it undergoes organic change, and lessens its variable constituent”\(^5\).

\[
P = P_o g \left( w - \bar{w} \right) t
\]

where \(g\) is a sign preserving function such that \(\text{sign } g = \text{sign } \left( w - \bar{w} \right)\) and \(\bar{w}\) is a positive constant such that\(^6\) \(1 - \bar{w}m > 0\).

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5. Marx, op. cit., p. 630
\[
\frac{dw}{dt} = f \left( \frac{meut}{h} K - P \right), \text{ where } f \text{ is the sign preserving function such that } \text{sign} f = \text{sign} \left( \frac{meut}{h} K - P \right).
\]

such that \( s' \) is the average rate of saving with respect to non-wage-income. Marx often assumes \( s' = 1 \) for illustrative purposes.

Thus

\[
\begin{bmatrix}
\frac{d(\log K)}{dt} > \frac{s}{h} \frac{meut}{h} K < P \\
\frac{d(\log K)}{dt} < \frac{s}{h} \frac{meut}{h} K > P
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\frac{d(\log K)}{dt} > \frac{d(\log P)}{dt} = 0
\end{bmatrix}
\]

\( w = \bar{w} \)

In a situation when the 'composition of capital' remains constant, \( u = 0 \), if initially \( w = \bar{w}, \frac{m}{h} K_o = P_o \) and since,

\[
\begin{bmatrix}
\frac{d(\log K)}{dt} > \frac{d(\log P)}{dt} = 0
\end{bmatrix}
\]

\( t = 0 \)

the subsequent situation will be one of \( \frac{m}{h} K > P \) which leads to increasing \( \frac{d(\log P)}{dt} \) and decreasing \( \frac{d(\log K)}{dt} \) consequent upon increasing \( w \).

This will ultimately lead to

\[
\begin{bmatrix}
\frac{d(\log P)}{dt} > \frac{d(\log K)}{dt}
\end{bmatrix}
\]

\( w > \bar{w} \) at \( \frac{m}{h} K = P \).

Thereafter, the situation will be one of \( \frac{m}{h} K < P \) which is a situation of redundancy of labour (of "involuntary" unemployment). The process,

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7. Marx, op. cit., ch. XXV.
then, is reversed as one of decreasing \( \frac{d (\log P)}{dt} \) and increasing \( \frac{d (\log K)}{dt} \) consequent upon decreasing \( w \). This will lead to
\[
\left[ \frac{d (\log P)}{dt} < \frac{d (\log K)}{dt} \right] \quad \text{at} \quad \frac{m}{h} K = P.
\]

Thus, the cyclic process continues. "On the other hand, accumulation slackens in consequence of the rise in the price of labour, because the stimulus of gain is blunted. The rate of accumulation lessens; but with its lessening, the primary cause of that lessening vanishes, i.e., the disproportion between capital and exploitable labour-power. The mechanism of the process of capitalist production removes the very obstacles that it temporarily creates. The price of labour falls again to a level corresponding with the needs of the self-expansion of capital, whether the level be below, the same as, or above the one which was normal before the rise of wages took place".9 The situation is thus, not different from the phenomenon corresponding to business cycle in Harrod-Domar model, except that in Harrod-Domar model there are other possibilities as well, that of constant rate of growth of income, redundancy of capital or redundancy of labour because of the assumption of constant rate of growth of population10. With the other two assumption about \( u \), the situation is either cycles of full employment and involuntary unemployment or one of increasing involuntary un-employment. "The labouring population", says Marx11, "is turned into a relative surplus-population; and it does this to an always increasing extent."

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